Scaling Dimensions in AdS/QCD and the Gluon Field Strength Propagator

Joshua W. Powell*
Department of Physics, Duke University, Durham NC 27708
(Dated: September 18, 2012)

We derive the scaling dimension of boundary operators in the AdS/CFT correspondence using a path integral treatment of the boundary-to-boundary propagators of their dual fields. We then apply the same technique to AdS/QCD where scaling dimensions are anomalous. In particular, we compute the two-point correlation function of gluon field strength operators, for which it is prerequisite to know the flow of the anomalous scaling dimension. The results are in very good agreement with quenched lattice QCD data, thus confirming the functional form of the scaling dimension.

In this paper, we solve two closely related problems. First, we extend the formula for the scaling dimension of boundary operators known from gauge/gravity dualities between scale invariant theories to a correspondence between theories without scale invariance. Specifically, we work with the extension of AdS/CFT to AdS/QCD, which has a warped background that breaks conformality and gives anomalous dimensions to the operators. The scaling dimensions are expressed by a path integral treatment of propagators of the dual fields in the bulk. Path integrals in curved spaces are subtle and we calibrate ours using what is known in the AdS/CFT correspondence. Afterwards we apply the technique to a specific warped but asymptotically-AdS background often used in AdS/QCD calculations. Second, we calculate the two-point function of the gluon field strength operator and compare with results computed in quenched lattice QCD. In order to carry out our calculation, we must use the formula for the flow of the field strength's anomalous dimension into the infrared. In turn, the good agreement between our calculation of the propagator and what is calculated on the lattice lends strong support to the formula for the scaling dimension.

The AdS/CFT correspondence between $\mathcal{N}=4$ supersymmetric Yang-Mills theory in four dimensions and supergravity on the background $\mathrm{AdS}_5\times S^5$, has opened up a new venue for probing strongly coupled behavior of quantum field theories [1]. Since its appearance, there has been hope that closely related techniques can be used to study strongly coupled physics in theories without supersymmetry or conformal symmetry. In particular, there has been a great deal of effort to find a gravitational theory dual to QCD in order to study its low energy, nonperturbative regime. Some attempts to achieve this goal begin, as in the AdS/CFT correspondence itself, with a string theory in a background configuration of branes [2, 3]. Others instead posit a gravitational background presumed to encapsulate the scaling behavior of QCD. The validity of this approach is then tested by using the model to compute various quantities in the gravitational theory which are compared to experiment results or other calculations, often performed using lattice QCD. Agreement between the two is taken as support for the original assumption. This approach to studying low energy QCD is known as AdS/QCD [4–7]. In this paper, it is the framework we shall adopt. One shortcoming of this approach is that it does not address the dynamical origin of the backgrounds considered.

One of the earliest results of the AdS/CFT correspondence was a computation of the anomalous scaling dimensions of boundary operators [8, 9]. This was achieved by computing the Green functions of bulk fields dual to the operators. Let us generalize slightly to a d-dimensional boundary theory and focus on R-symmetry singlet operators, meaning we ignore the S^5 coordinates in the bulk spacetime. Now the dual gravitational theory effectively lives on the background AdS_{d+1}. We use coordinates in which the metric takes the form

$$ds^2 = g_{mn}dx^m dx^n = \frac{R^2}{z^2}\left(dz^2 + \delta_{\mu\nu}\, dx^\mu dx^\nu\right). \qquad (1)$$

Note that we have Euclideanized the metric. A $(d-p)$-form operator on the boundary is dual to a p-form field in the bulk. Absent any background fields to provide special reference directions, the index structure of the Green function must be trivial. We will write $G_{\mathcal{AB}}(x,y) = \delta_{\mathcal{AB}}\, G_p(x,y)$, where $\mathcal{A}$ and $\mathcal{B}$ are generic indices appropriate for a p-form and

$$G_p(x,y) = \langle x|(\Box_{(p)} + m^2)^{-1}|y\rangle\,, \qquad (2)$$

where $\Box_{(p)}$ is the Hodge Laplacian on a p-form field [10, 11]. As we will see later, the presence of Wilson lines provides a background, and it is only when we neglect its background that we keep our trivial index structure. In Ref. [8, 9], computation of $G_p(x,y)$ hinged on the ability to solve the relevant differential equations directly, which is not possible in the confining background we will consider later. Instead, we will use a semiclassical evaluation of the path integral [12–14]. The Green function is given by

* jwp14@phy.duke.edu

$$G_p(x,y) = \int_{z(0)=y}^{z(1)=x} [dz(\lambda)\, de(\lambda)]\, \exp\left(-\frac{1}{2}\int_0^1 \left[e^{-1}\dot{z}^m \dot{z}^n g_{mn}(z) + e\,(m^2 - \kappa_{d,p}\mathcal{R})\right] d\lambda \right). \tag{3}$$

Here $z^m(\lambda)$ is a parametrization of the path followed by the particle, e is an einbein field associated with the parametrization, $d+1$ is the dimensionality of the spacetime and $\mathcal{R} = -d(d+1)/R^2$ is the Ricci scalar curvature. As emphasized in Ref. [15], there is an ordering ambiguity in going from the classical action to the quantum action in a curved background. In a spacetime covariant formulation, the ambiguity allows a curvature term in the quantum action with a coefficient that can only be be determined by imposing some other condition. By dimension counting and analyticity, the curvature term must be the Ricci scalar. Its coefficient $\kappa_{d,p}$ is a dimensionless constant which can depend only on the spacetime dimension and degree p of the form field. We will determine the value of $\kappa_{d,p}$ by reproducing the classical propagator in AdS_{d+1}. In the warped but asymptotically AdS_{d+1} background, the same value is found by matching results for the propagator between two very close boundary points because the worldline stays in the near boundary region. This value then holds everywhere.

We make the small fluctuation approximation by expanding Eq. (3) around its classical solution and treating the fluctuations only to quadratic order [14], which yields

$$G(x,y) = D_{\mathrm{VM}}(x,y)^{1/2}\, \exp[-S_{\mathrm{cl}}(x,y)]\,. \tag{4}$$

$S_{\mathrm{cl}}(x,y)$ is the classical action of the particle propagating from x to y and $D_{\mathrm{VM}}(x,y)$ is the Van Vleck–Morette determinant (VMD), whose square root is the fluctuation determinant [14, 16]. The path integral over z depends only on $s = \int_0^1 e\, d\lambda$, so that the path integral over the einbein reduces to a simple integral over s. Let z_{cl} denote the classical solution of the equations of motion connecting x to y and let $\sigma(x,y)$ be the proper distance of this path. Then the classical action term in Eq. (4) becomes

$$\begin{aligned} \exp[-S_{\mathrm{cl}}(x,y)] &= \int_0^\infty \frac{ds}{(2\pi s)^{(d+1)/2}}\, \exp\left(-\frac{\sigma(x,y)^2}{2s} - \frac{s}{2}(m^2 - \kappa_{d,p}\mathcal{R})\right) \\ &= 2\left((m^2 - \kappa_{d,p}\mathcal{R})\Big/\sigma(x,y)^2 \right)^{(d-1)/4} K_{(d-1)/2}\left(\sigma(x,y)\sqrt{m^2 - \kappa_{d,p}\mathcal{R}}\right). \end{aligned} \tag{5}$$

The fluctuation determinant $D_{\mathrm{VM}}(x,y)^{1/2}$ is

$$\begin{aligned} D_{\mathrm{VM}}(x,y)^{1/2} &= \int_{\zeta(0)=0}^{\zeta(1)=0} [d\zeta(\lambda)] \exp\left(-\frac{1}{2s}\int_0^1 \left[g_{mn}(z_{\mathrm{cl}})\dot{\zeta}^m\dot{\zeta}^n - \mathcal{R}_{mpnq}\zeta^m \dot{z}^p_{\mathrm{cl}}\zeta^n \dot{z}^q_{\mathrm{cl}}\right] d\lambda \right) \\ &= \int_{\zeta(0)=0}^{\zeta(1)=0} [d\zeta(\lambda)] \exp\left(\frac{1}{2s}\int_0^1 \zeta^m\, (g_{mn}\partial_\lambda^2 + M_{mn})\zeta^n\, d\lambda\right), \end{aligned} \tag{6}$$

for $\mathcal{R}_{mrns}$ the Riemann curvature tensor and $M_{mn} = \mathcal{R}_{mrns}\dot{z}^r_{\mathrm{cl}}\dot{z}^s_{\mathrm{cl}}$. The action for the fluctuation field ζ comes from the second variation of the path length around the classical solution. The equations of motion of ζ are known as the geodesic deviation equation and the Jacobi equation. The particle's trajectory has been decomposed as

$$z^m(\lambda) = z^m_{\mathrm{cl}}(\lambda) + \zeta^m(\lambda)\,, \tag{7}$$

with the constraint $g_{mn}(z_{\mathrm{cl}})\dot{z}^m_{\mathrm{cl}}\zeta^n = 0$ so that fluctuations which are always orthogonal classical solution. This means the fluctuation field has only d degrees of freedom.

AdS_{d+1} has Riemann curvature tensor

$$\mathcal{R}_{mrns} = -\frac{1}{R^2}\,(g_{mn}g_{rs} - g_{ms}g_{nr})\,. \tag{8}$$

Taking λ to be an affine parameter along the geodesic $z^m_{\mathrm{cl}}(\lambda)$, the tangent vector $\dot{z}^m_{\mathrm{cl}}(\lambda)$ has constant length,

$$g_{mn}(z_{\mathrm{cl}})\dot{z}^m_{\mathrm{cl}}(\lambda)\dot{z}^n_{\mathrm{cl}}(\lambda) = \sigma(x,y)^2\,. \tag{9}$$

Using this fact and the orthogonality relationship $g_{mn}\zeta^m\dot{z}^n_{\mathrm{cl}} = 0$, we find

$$M_{mn} = -g_{mn}\sigma(x,y)^2/R^2\,. \tag{10}$$

Eq. (6) can be expressed using functional determinants and computed exactly,

$$D_{\rm VM}(x,y)^{1/2} = \left(\frac{\operatorname{Det}\det{}'(-g_{mn}\partial_\lambda^2 - M_{mn})}{\operatorname{Det}\det{}'(-g_{mn}\partial_\lambda^2)}\right)^{-1/2} = \left(\frac{\sigma(x,y)/R}{\sinh(\sigma(x,y)/R)}\right)^{d/2}. \tag{11}$$

Here $\det{}'$ denotes the determinant over vector indices m and n only in the subspace orthogonal to the classical solution. This means $\det{}'$ excludes a zero mode from the computation of the determinant of M_{mn}. Det denotes the functional determinant. There are numerous ways to compute the functional determinant but we will evaluate it using the Gel'fand-Yaglom Theorem [17, 18], because it is used in a less trivial case later. First expand

$$\left(\frac{\operatorname{Det}(-g_{mn}\partial_\lambda^2 - g_{mn}\sigma(x,y)^2/R^2)}{\operatorname{Det}(-g_{mn}\partial_\lambda^2)}\right) = \left(\frac{\operatorname{Det}(-\partial_\lambda^2 - \sigma(x,y)^2/R^2)}{\operatorname{Det}(-\partial_\lambda^2)}\right)^d, \tag{12}$$

using the diagonality of the metric and cancelled a common normalization between the numerator and denominator. Even though the theory lives in $d+1$ dimensions the exponent is only d, reflecting the fact that only fluctuations tangent to the classical solution are considered. The Gel'fand-Yaglom Theorem states that the normalized determinant of an operator $-\partial_\lambda^2 - V(\lambda)$ acting on space of functions supported on the interval $0 \leq \lambda \leq 1$ with vanishing boundary values is

$$\frac{\operatorname{Det}(-\partial_\lambda^2 - V(\lambda))}{\operatorname{Det}(-\partial_\lambda^2)} = u(1)^{-1}, \tag{13}$$

where

$$(\partial_\lambda^2 + V(\lambda))u(\lambda) = 0, \tag{14}$$

with boundary conditions $u(0) = 0$ and $\dot{u}(0) = 1$. Eq. (11) follows immediately for the trivial case of a constant $V = -\sigma(x,y)^2/R^2$.

Using Eqs. (5) and (11) in Eq. (4) shows that the semiclassical approximation to the Green function takes the form

$$G(x,y) = \mathcal{N} K_{(d-1)/2}\left(\sigma(x,y)\sqrt{m^2 - \kappa_{d,p}\mathcal{R}}\right) \times \left(\frac{\sigma(x,y)/R}{\sinh(\sigma(x,y)/R)}\right)^{d/2}\left(\frac{\sqrt{m^2-\kappa_{d,p}\mathcal{R}}}{\sigma(x,y)}\right)^{(d-1)/2}. \tag{15}$$

$\mathcal{N}$ is a normalization factor and $K_{(d-1)/2}$ is a modified Bessel function. To compute the boundary-to-boundary propagator, put $x = (x^\mu, \varepsilon)$ and $y = (y^\mu, \varepsilon)$, for ε positive but infinitesimal. The geodesic distance is

$$\sigma(x,y) = R \ln \frac{|x^\mu - y^\mu|^2}{\varepsilon^2} + O(\varepsilon), \tag{16}$$

which diverges as ε tends to 0. This pushes the Bessel and sinh functions into their asymptotic regimes, with $\sinh x \approx e^x/2$ and $K_{(d-1)/2}(x) \approx \sqrt{\pi/2x}\, e^{-x}$ for any d. The asymptotic behavior of the Bessel function, the normalization factor in $\exp(-S_{\rm cl})$ and the fluctuation determinant combine to eliminate dependence on $\sigma(x,y)$ everywhere except in the exponent. We regularize $\sigma(x,y)$ at both endpoints with some scale μ by setting $\sigma(x,y) = \sigma_{\rm reg}(x,y) - 2R\ln(\mu\varepsilon)$ and absorbing the divergent term into the normalization, $\mathcal{N} \to \mathcal{N}'(\mu)$. The resulting Green function is

$$G_{\rm reg}(x,y) = \mathcal{N}' \exp(-\Delta\,\sigma_{\rm reg}(x,y)/R) = \frac{\mathcal{N}'}{|x-y|^{2\Delta}}, \tag{17}$$

with exponent

$$\Delta = \frac{d}{2} + \sqrt{(MR)^2 - \kappa_{d,p}R^2\mathcal{R}}. \tag{18}$$

The first term on the right comes from $D_{\rm VM}(x,y)^{1/2}$ and the second term comes from $\exp[-S_{\rm cl}(x,y)]$. By taking

$$\kappa_{d,\,p} = \frac{1}{d(d+1)}\left(\frac{d}{2} - p\right)^2, \tag{19}$$

we recover the scaling behavior familiar from AdS/CFT [8, 9]. We can make the decomposition

$$-\kappa_{d,p}\mathcal{R} = \frac{\mathcal{R}}{d(d+1)}\,p(d-p) - \frac{d-1}{4d}\,\mathcal{R} - \frac{\mathcal{R}}{4d(d+1)}. \tag{20}$$

The first term on the right is carried over from the classical field Lagrangian for p-form fields, and comes from the Hodge Laplacian [10, 11]. The second term is the conformal coupling to the curvature. The third term is the path integral measure Jacobian $\frac{1}{4}R^{-2}$ [19]. If the background were instead a sphere of radius R, this term would be $-\frac{1}{4}R^{-2}$ [20]. In general, it adjusts for the difference in normalization of the path measure between flat and curved spacetimes.

In Ref. [21] the Green function is also computed in the semiclassical approximation. The author notes how the two-point function of a field in AdS behaves as though Δ/R appears in place of the mass m in the field's classical action. Here, we have shown how a path integral computation of the two-point function can generate the needed terms to convert m into the correct full expression for Δ/R. The path integral technique gives a robust understanding of how Δ emerges from kinematics in a more general curved background, which will allow us to understand how its value is modified in a warped background.

We now introduce some machinery necessary for the computation of the gluon field strength two-point function. Yang-Mills field strength operators are not gauge invariant, and the field strength two-point function requires a Wilson loop to form a gauge invariant quantity.

We define a loop current on a closed contour in spacetime $\mathcal{C}$, parametrized by $y(\lambda)$, by

$$j^\mu_{\mathcal{C}}(x) = \int_{\mathcal{C}} \dot{y}^\mu(\lambda)\, \delta^{(4)}(x - y(\lambda))\, d\lambda\,. \tag{21}$$

The current enters the functional integral via the factor $\exp(-S_{\rm int}[A,j]) = \exp(-\int A\cdot j\, dx)$, the standard minimal coupling of a conserved current to a gauge field. In non-Abelian theories, the exponential is understood to be path-ordered. We use a shorthand notation for the expectation value of an operator $\mathcal{O}$ in the presence of the loop current on $\mathcal{C}$,

$$\langle\!\langle \mathcal{O} \rangle\!\rangle_{\mathcal{C}} = N_c^{-1} \int [dA]\, e^{-S_{\rm YM}}\, {\rm tr}\, \mathcal{P}\{\mathcal{O}\, \exp(-\oint_{\mathcal{C}} A(x)\cdot dx)\}\,. \tag{22}$$

The trace is over gauge degrees of freedom and $\mathcal{P}$ denotes path ordering. The normalization is chosen so that the value of the partition function $Z[j_{\mathcal{C}}] = \langle\!\langle 1 \rangle\!\rangle_{\mathcal{C}} \to 1$ as $\mathcal{C}$ tends to a point. Note that the Wilson loop defined on the contour $\mathcal{C}$ is precisely $Z[j_{\mathcal{C}}]$. The two-point function of field strength operators is

$$G^{(FF)}_{\mu\nu\rho\sigma}(x,y|\mathcal{C}) = \langle\!\langle F_{\mu\nu}(x) F_{\rho\sigma}(y) \rangle\!\rangle_{\mathcal{C}} / \langle\!\langle 1 \rangle\!\rangle_{\mathcal{C}}\,. \tag{23}$$

Its value depends on $\mathcal{C}$ and gauge invariance requires that the points x and y must lie on $\mathcal{C}$. Here and throughout the rest of the paper, μ and ν refer to boundary directions, i.e. all directions except the radial direction z.

First we review how to compute $\langle\!\langle 1 \rangle\!\rangle_{\mathcal{C}}$. The non-Abelian Stokes' Theorem [22] states

$$\begin{aligned}&\int [dA]\, e^{-S_{\rm YM}}\, \mathcal{P}{\rm exp}\left(-\oint_{\mathcal{C}} A(z)\cdot dz\right)\\ &\quad = \int [dA]\, e^{-S_{\rm YM}}\, \mathcal{P}{\rm exp}\left(-\oint_{\mathcal{S}} F\cdot d^2 a\right).\end{aligned} \tag{24}$$

Here $\mathcal{C}$ is the boundary of surface $\mathcal{S}$ with coordinates ξ and differential area 2-form $d^2a^{\mu\nu}(\xi)$, and $F\cdot d^2 a = \frac{1}{2} F_{\mu\nu} d^2 a^{\mu\nu}(\xi)$. This latter expression is discretized and computed in lattice QCD. On the lattice, path integrals over different configurations of the gauge field correspond to sums over states with different plaquettes turned on. With no dynamical sources of charge around, the plaquettes which are turned on must span a surface $\mathcal{S}$ ending on contour $\mathcal{C}$. In the strong coupling limit, the minimal area surface dominates [23].

The computation of $\langle\!\langle 1 \rangle\!\rangle_{\mathcal{C}}$ can also be performed in AdS/CFT [24] and using AdS/QCD [6]. The strongly coupled gauge dynamics in the Wilson loop is dual to a Nambu-Goto string with endpoints on the contour of the Wilson loop. The string worldsheet $X^m(\xi)$ has boundary $\mathcal{C}$ on the conformal boundary of the $(d+1)$-dimensional spacetime, and

$$\langle\!\langle 1 \rangle\!\rangle_{\mathcal{C}} = \exp(-S_{NG}[X])\,. \tag{25}$$

Recall the Nambu-Goto action is given by

$$S_{\rm NG}[X] = \frac{1}{2\pi\alpha'} \int d^2\xi\, \sqrt{\det g_{mn}(X) \partial_a X^m \partial_b X^n}\,. \tag{26}$$

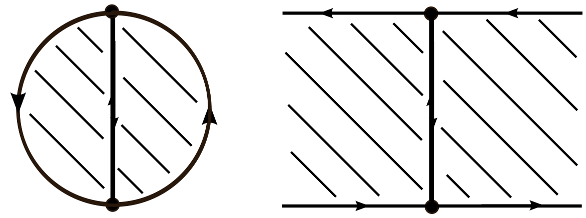

FIG. 1. Two possible contours $\mathcal{C}$. The contour on the right is the one used in our computations.

We work in the asymptotically Euclidean AdS$_{d+1}$ "metric wall" background of Ref. [6], with metric

$$ds^2 = g_{mn} dx^m dx^n = e^{4\Lambda^2 z^2}\, \frac{R^2}{z^2}\, (dz^2 + \delta_{\mu\nu}\, dx^\mu dx^\nu)\,. \tag{27}$$

In Ref. [25], this background is compared with alternative metrics in its ability to compute rectangular Wilson loops, and is shown to provide the best agreement with lattice computations of the same. We determine our parameters by reproducing lattice results for the Cornell potential between a heavy quark-antiquark pair modeled as a rectangular line current. Doing so yields $\Lambda \approx 330\,$MeV and the dimensionless string tension $\tau = R^2/2\pi\alpha' \approx 0.1836$. The numerator of Eq. (23) is the expectation value of two field strength operators inserted along the Wilson loop along the contour $\mathcal{C}$ in the background generated by the rest of $\mathcal{C}$. The insertion of $F_{\mu\nu}$ at a point along $\mathcal{C}$ can be achieved by taking the area derivative of the contour at this point [26, 27]. In essence, this amounts to varying the contour $\mathcal{C}$ not with the typical Dirac δ function spike but with a "keyboard variation." On the lattice, this variation is made by adding one extra plaquette in the μ-ν plane at the appropriate point along the contour, modifying the contour $\mathcal{C}$ to $\mathcal{C} + \delta\mathcal{C}$. We can expand the value of the Wilson loop in powers of this small change, which gives at first order

$$\langle\!\langle 1 \rangle\!\rangle_{\mathcal{C}+\delta\mathcal{C}} = Z[j_{\mathcal{C}} + \delta j^{\mu\nu}(x)] \approx \langle\!\langle 1 - a^{\mu\nu} F_{\mu\nu}(x) \rangle\!\rangle_{\mathcal{C}}\,. \tag{28}$$

Here $a^{\mu\nu}$ is the oriented area of the extra plaquette added at x. Two derivatives insert two field strength operators

$$\langle\!\langle F_{\mu\nu}(x) F_{\rho\sigma}(y) \rangle\!\rangle_{\mathcal{C}} = \left.\frac{\delta^2 Z[j]}{\delta j^{\mu\nu}(x) \delta j^{\rho\sigma}(y)}\right|_{j=j_{\mathcal{C}}}\,. \tag{29}$$

In gauge/gravity dualities, conserved currents on the boundary are source terms of gauge fields in the bulk. The keyboard-type variational second derivative in Eq. (29) is the two-point function of the field strength in the gauge field background with source supported along the contour $\mathcal{C}$. It is shown in Ref. [28] for the AdS/CFT correspondence that when $\mathcal{C}$ is a straight infinite line

$$\langle\!\langle F_{\mu\nu}(x) F_{\rho\sigma}(y) \rangle\!\rangle_{\mathcal{C}} \propto 1/|x-y|^4\,, \tag{30}$$

as expected by the conformality of $\mathcal{N} = 4$ supersymmetric Yang-Mills. For theories without conformal invariance, power counting alone does not determine the

two-point function. We extend the treatment of the field strength two-point function presented in Ref. [28] to nonconformal backgrounds. The two-point function of a $(d-p)$-form operator on the boundary corresponds to the propagation of a particle transforming as a p-form, which we take to have mass m, in the dual gravitational theory. In general, form fields have gauge symmetries so we will assume for generality that the particle moves in a background gauge field A. Its Green function is [12]

$$G_{\mathcal{AB}}(x,y|A) = \langle x|[(D[A]^2+m^2)\,\mathbf{1}+F[A]\cdot\mathbf{\Sigma}]^{-1}_{\mathcal{AB}}|y\rangle\,, \tag{31}$$

where $\mathbf{\Sigma}^{mn}$ is the generator of spacetime rotations in the m-n plane in the p-form representation, which we take to have generic indices $\mathcal{A}$, $\mathcal{B}$. $D[A]$ and $F[A]$ are the covariant derivative and field strength computed using the background field A only. We can write the Green function in terms of a path integral [12],

$$G_{\mathcal{AB}}(x,y|A) = \int_{z(0)=y}^{z(1)=x} [dz(\lambda)\,de(\lambda)]\,\exp\left(-\frac{1}{2}\int_0^1\left[e^{-1}\dot{z}^m\dot{z}^n g_{mn}(z)+e\,(m^2-\kappa_{d,p}\mathcal{R}(z))\right]d\lambda\right) \times\mathcal{P}\mathrm{exp}\left(-\int_0^1 A_m(z)\dot{z}^m d\lambda\right)\left[\mathcal{P}\mathrm{exp}\left(-\int_0^1 F(z)\cdot\mathbf{\Sigma}\,e\,d\lambda\right)\right]_{\mathcal{AB}}. \tag{32}$$

The field strength two-point function needs the Wilson loop to be gauge-invariant. In the bulk theory, the Wilson loop is dual to a Nambu-Goto string. Therefore, gauge invariance dictates the worldline of the field strength propagator must lie within the Nambu-Goto worldsheet.

The parallel transporter $\mathcal{P}\mathrm{exp}(-\int A\cdot dz)$ in the field strength propagator in Eq. (3) transforms in the adjoint representation of the gauge group. It is realized in lattice simulation as coincident fundamental and antifundamental parallel transporters [29, 30]. The antifundamental parallel transporter completes half the line current $j_{\mathcal{C}}$, and the fundamental parallel transporter completes the other half, as shown in Fig. 1. Thus the worldline of the field strength propagator splits the worldsheet into two subregions.

In the computations of Ref. [31], the field strength correlator is computed as a weighted sum over the results obtained with different Wilson loop contours. For this reason, it is not entirely clear exactly which contour $\mathcal{C}$ we should use. Two possibilities are shown in Fig. 1. However, there should not be strong dependence on the exact contour $\mathcal{C}$ in the AdS/QCD calculations, as long as the worldsheet it bounds is entirely in the near-boundary region of the bulk for small r and can extend up to $z_\Lambda=(2\Lambda)^{-1}$ at large r. We use the contour shown on the right side of Fig. 1. We take x and y to be separated only in the time direction t by a boundary distance of $|x-y|=r$. The profile is translationally invariant in some boundary direction perpendicular to t. The cross section in the t direction is given by $z_{\mathrm{cl}}(t)$ which satisfies [6]

$$\frac{e^{4\Lambda^2 z_{\mathrm{cl}}(t)^2}}{z_{\mathrm{cl}}(t)^2\sqrt{1+z'_{\mathrm{cl}}(t)^2}}=\frac{e^{4\Lambda^2 z_m}}{z_m^2}\,, \tag{33}$$

where z_m is the maximum value the string's profile assumes, and is determined implicitly by

$$r/2=\int_0^{z_m}dz\left(\frac{z_m^4}{z^4}\,e^{8\Lambda^2(z_m^2-z^2)}-1\right)^{-1/2}. \tag{34}$$

The shape of this profile is shown for several values of r in Fig. 2. Notice that for very small r, a slight change in r results in a new profile which is just a rescaled version of the old profile. This is due to asymptotic conformality as z approaches 0. At large r, the string sits at $z=z_m\approx(2\Lambda)^{-1}$ for the majority of its extent in the boundary directions. Increases in r serve only to extend this portion. Such behavior is common to all choices of contour $\mathcal{C}$, with differences lying only in minor details of the exact profile.

There are two ways in Eq. (32) where the background field with source supported along the contour $\mathcal{C}$ enters the evaluation of $G^{(FF)}_{\mathrm{reg}}(x,y|\mathcal{C})$. First, the background field enters through the spin-dependent term

$$[\mathcal{P}\mathrm{exp}(-\textstyle\int F\cdot\mathbf{\Sigma}\,e\,d\lambda)]_{\mathcal{AB}}=\delta_{\mathcal{AB}}+\cdots\,. \tag{35}$$

We compute only the term proportional to $\delta_{\mathcal{AB}}$. For a 2-form, the generic index $\mathcal{A}=[\mu\nu]$, so we are keeping only the term proportional to

$$\delta_{[\mu\nu][\rho\sigma]}=\delta_{\mu\rho}\delta_{\nu\sigma}-\delta_{\mu\sigma}\delta_{\nu\rho}\,, \tag{36}$$

which is called $D_\perp(x,y)$ in Ref. [31].

The other appearance of the background field is through

$$\mathcal{P}\mathrm{exp}(-\textstyle\oint A\cdot dx)=\mathcal{P}\mathrm{exp}(-\int F\cdot d^2a)\,. \tag{37}$$

In the bulk theory, this becomes part of the Nambu-Goto string action. To account for its influence, we find the geometry of the string worldsheet and particle's worldline

which minimize their total action subject to the constraint that the worldline stay within the worldsheet. We make the approximation that the balance they strike is exactly the string's profile. It is worth repeating for emphasis that the point particle's trajectory follows the Nambu-Goto string's profile and not the minimum distance geodesic connecting the endpoints. To see this, consider the quantities

$$\langle\!\langle F_{\mu\nu}(x)F_{\rho\sigma}(y)\rangle\!\rangle_{\mathcal{C}} = \exp(-S_{\rm NG}[X(\mathcal{C})] - S_{\rm cl}(x,y)) \quad (38)$$
$$\langle\!\langle 1\rangle\!\rangle_{\mathcal{C}} = \exp(-S_{\rm NG}[X(\mathcal{C})]) \quad (39)$$

Although the Nambu-Goto action cancels out of $G^{(FF)}_{\mu\nu\rho\sigma}(x,y|\mathcal{C}) = \langle\!\langle F_{\mu\nu}(x)F_{\rho\sigma}(y)\rangle\!\rangle_{\mathcal{C}}/\langle\!\langle 1\rangle\!\rangle_{\mathcal{C}}$ in the term $\propto \delta_{[\mu\nu][\rho\sigma]}$, it still affects the geometry of the minimal action state. It determines it completely in our approximation. There is interaction energy which will cause it to deviate from the case without field strength operator insertions along the boundary, but the deviation from our approximation is very small. So we must only compute the action of the field strength propagating along the string's profile.

When we drop the spin-dependent coupling term in Eq. (32), the propagator once again takes the form in Eq. (4). But now the curvature terms vary with position

$$\begin{aligned}\exp[-S_{\rm cl}(x,y)] &= \int_0^\infty \frac{ds}{(2\pi s)^{(d+1)/2}}\,\exp\left(-\frac{\sigma(x,y)^2}{2s} - \frac{s}{2}\int_0^1 [m^2 - \kappa_{d,p}\mathcal{R}(z_{\rm cl}(\lambda))]d\lambda\right) \quad (40)\\ &= 2\left(\int_0^1[m^2-\kappa_{d,p}\mathcal{R}(z_{\rm cl}(\lambda))]d\lambda\Big/\sigma(x,y)^2\right)^{(d-1)/4} K_{(d-1)/2}\left(\sigma(x,y)\sqrt{\int_0^1[m^2-\kappa_{d,p}\mathcal{R}(z_{\rm cl}(\lambda))]\,d\lambda}\right).\end{aligned}$$

and

$$\begin{aligned}D_{\rm VM}(x,y)^{1/2} &= \int_{\zeta(0)=0}^{\zeta(1)=0}[d\zeta(\lambda)]\exp\left(-\frac{1}{2s}\int_0^1\left[g_{mn}(z_{\rm cl})\dot\zeta^m\dot\zeta^n - \mathcal{R}_{mpnq}(z_{\rm cl})\zeta^m\dot z^p_{\rm cl}\zeta^n\dot z^q_{\rm cl}\right]d\lambda\right) \quad (41)\\ &= \int_{\zeta(0)=0}^{\zeta(1)=0}[d\zeta(\lambda)]\exp\left(\frac{1}{2s}\int_0^1\zeta^m\,[g_{mn}\partial_\lambda^2 + M_{mn}]\zeta^n\,d\lambda\right),\end{aligned}$$

The field strength operator is a 2-form, so for $d=4$ there is a simplification because $\kappa_{4,2}=0$. Moreover, its mass vanishes. This can be determined in the near boundary region, where $\Delta = 2$ and the AdS/CFT scaling dimension formula applies. However, we must first take the large $\sigma(x,y)$ limit of Eq. (40), keeping m finite as a regulator, and afterwards we can absorb m into the normalization. At the end of these manipulations, we find

$$\exp[-S_{\rm cl}(x,y)] \to \mathcal{N}\,\sigma(x,y)^{-d/2}\,. \quad (42)$$

$D_{\rm VM}(x,y)^{1/2}$ can be expressed using the determinant of the operator $g_{mn}\partial_\lambda^2 + M_{mn}$,

$$\left(\frac{{\rm Det}\,{\rm det}'(-g_{mn}\partial_\lambda^2 - M_{mn})}{{\rm Det}\,{\rm det}'(-g_{mn}\partial_\lambda^2)}\right)^{-1/2}. \quad (43)$$

In the background metric of Eq. (1), the curvature is constant and isotropic. This means $M_{mn}(z_{\rm cl}(\lambda))$ is a constant as in Eq. (11). This feature also holds to a very good approximation in the near-boundary region of the warped background in Eq. (27). Furthermore, for large $r=|x-y|$ the string's profile will sit for the majority of the time at $z_\Lambda = (2\Lambda)^{-1}$, and for these points the value of $M_{mn}(z_{\rm cl}(\lambda))$ varies extremely slowly. The variation of M_{mn} with λ in the intermediate region is also slow. The curvature tensor in Eq. (51) is bounded above in absolute value by the unwarped curvature tensor in Eq. (8). The curvature is taken to be small compared to the scale set by the string tension. So it is justified to expand in $(\partial_\lambda^2)^{-1}M_{mn}$, and we use the expansion of the determinant of nearly unit matrix, $\det(1+\varepsilon) = 1 + {\rm tr}\,\varepsilon + O(\varepsilon^2)$, to write [32]

$$\begin{aligned}&{\rm det}'(-g_{mn}\partial_\lambda^2 - M_{mn}) \quad (44)\\ &\quad= {\rm det}'(-g_{mn}\partial_\lambda^2)\times[1 + (\partial_\lambda^2)^{-1}{\rm tr}\,M] + O(M^2)\\ &\quad= (-\partial_\lambda^2)^d - (-\partial_\lambda^2)^{d-1}\,{\rm tr}\,M + O(M^2)\\ &\quad= (-\partial_\lambda^2 - d^{-1}{\rm tr}\,M)^d + O(M^2,\,\partial^2 M)\end{aligned}$$

We then drop the higher order terms. For an unwarped AdS metric, the above result is exact and the dropped

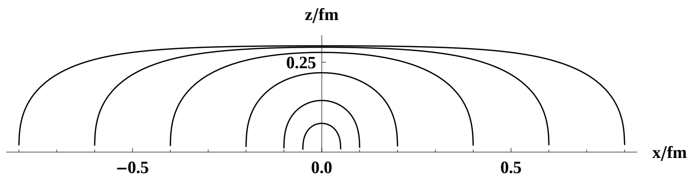

FIG. 2. String profiles with various values of r

terms in fact vanish. Next, use the relationship

$$D_{\mathrm{VM}}(x,y)^{1/2} = \left(\frac{\mathrm{Det}[(-\partial_\lambda^2 - d^{-1}\mathrm{tr}\,M_{mn})^d]}{\mathrm{Det}[(-\partial_\lambda^2)^d]}\right)^{-1/2} = \left(\frac{\mathrm{Det}(-\partial_\lambda^2 - d^{-1}\mathrm{tr}\,M_{mn})}{\mathrm{Det}(-\partial_\lambda^2)}\right)^{-d/2}. \tag{45}$$

This manipulation is valid here and in Eq. (12) because the functional determinant is a power of a single differential operator and not of the product of different differential operators [33]. The functional determinant is now readily computed using the Gel'fand-Yaglom theorem,

$$\frac{\mathrm{Det}(-\partial_\lambda^2 - d^{-1}\mathrm{tr}\,M_{mn})}{\mathrm{Det}(-\partial_\lambda^2)} = u(1)^{-1} \tag{46}$$

where $u(\lambda)$ satisfies

$$(\partial_\lambda^2 + d^{-1}\mathrm{tr}\,M_{mn})u(\lambda) = 0 \tag{47}$$

with boundary conditions $u(0) = 0$ and $\dot{u}(0) = 1$. Since we are taking M_{mn} to be small and slowly varying, we use a WKB approximation, resulting in

$$D_{\mathrm{VM}}^{1/2} \approx \left(\frac{\sinh(\int_0^\lambda \sqrt{-d^{-1}\mathrm{tr}\,M_{mn}(\lambda)}\,d\lambda')}{\sqrt{-d^{-1}\mathrm{tr}\,M_{mn}(0)}}\right)^{-d/2}. \tag{48}$$

To express $\sqrt{-d^{-1}\mathrm{tr}\,M_{mn}}$, we first introduce a frame field ${e_m}^a = (R/z){\delta_m}^a$ or equivalently ${e^m}_a = (z/R){\delta^m}_a$, so that $g_{mn} = {e_m}^a {e_n}^b \delta_{ab}$. We write

$$\dot{z}_{\mathrm{cl}}^m(\lambda) = \sigma(x,y)\,{e^m}_a\,\hat{t}^{\,a}(\lambda)\,, \tag{49}$$

where $\hat{t}^{\,a}(\lambda)$ is the tangent along the geodesic normalized in the local rest frame, $\delta_{ab}\,\hat{t}^{\,a}(\lambda)\hat{t}^{\,b}(\lambda) = 1$. For notational convenience, we introduce $\theta(\lambda)$ which satisfies

$$\cos^2\theta(\lambda) = \hat{t}^{\,x}(\lambda)^2 \quad \text{and} \quad \sin^2\theta(\lambda) = \hat{t}^{\,z}(\lambda)^2\,. \tag{50}$$

The Riemann tensor of the metric in Eq. (27) has components

$$\begin{aligned}\mathcal{R}_{\mu\nu\rho\sigma} &= -\frac{1}{R^2}\,e^{-4\Lambda^2 z^2}\,(1-4\Lambda^2 z^2)^2\,(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})\,,\\ \mathcal{R}_{\mu z\nu z} &= -\frac{1}{R^2}\,e^{-4\Lambda^2 z^2}\,(1+4\Lambda^2 z^2)\,g_{\mu\nu}g_{zz}\,,\end{aligned} \tag{51}$$

and all others vanish except those related by symmetries of the Riemann tensor to the two components above. We find that

$$\sqrt{-d^{-1}\mathrm{tr}\,M_{mn}(z_{\mathrm{cl}}(\lambda))} = \frac{1}{R}\,e^{-2\Lambda^2 z^2}\sqrt{(1-d^{-1})[\cos^2\theta(1-4\Lambda^2 z^2)^2 + \sin^2\theta(1+4\Lambda^2 z^2)] + d^{-1}(1+4\Lambda^2 z^2)}\,, \tag{52}$$

where $z = z_{\mathrm{cl}}^z(\lambda)$ is the radial coordinate at a point along the classical path.

As x and y approach the boundary, sinh assumes its asymptotic form and we arrive at a simplification like in Eq. (17), so that the Green function takes the form

$$G^{(FF)}(x,y|\mathcal{C}) = \mathcal{N}\exp[-\Delta\cdot\sigma(x,y)/R]\,, \tag{53}$$

where Δ is defined by

$$\Delta(z_{\mathrm{cl}}(\lambda)) = R\,\frac{d}{2}\sqrt{-d^{-1}\,\mathrm{tr}\,M_{mn}(z_{\mathrm{cl}}(\lambda))} \tag{54}$$

and

$$\sigma\cdot\Delta/R = \frac{\sigma(x,y)}{R}\int_0^1 \Delta(z_{\mathrm{cl}}(\lambda))\,d\lambda\,. \tag{55}$$

Using an affine parameter in the expressions above allows the factorization of the distance $\sigma(x,y)$ out of the integral in Eq. (55). For a generic parametrization of the classical solution with parameter ξ, the quantity

$$\begin{aligned}\sigma\cdot\Delta/R &= \int_{\xi_x}^{\xi_y}\frac{d}{2}\sqrt{-d^{-1}\,\mathrm{tr}\,M(z_{\mathrm{cl}}(\xi))}\,\|dz_{\mathrm{cl}}(\xi)\|\\ &\equiv \frac{1}{R}\int_{\xi_x}^{\xi_y}\Delta(z_{\mathrm{cl}}(\xi))\,\|dz_{\mathrm{cl}}(\xi)\|\,,\end{aligned} \tag{56}$$

where $\|dz_{\mathrm{cl}}(\xi)\| = \sqrt{g_{mn}\,dz_{\mathrm{cl}}^m(\xi)dz_{\mathrm{cl}}^n(\xi)}$, $z_{\mathrm{cl}}(\xi_x) = x$ and $z_{\mathrm{cl}}(\xi_y) = y$. We will perform calculations using the t-coordinate parametrization of the classical solution given

in Eq. (33), for which

$$\sigma \cdot \Delta / R = \int_{-r/2}^{r/2} \Delta(z_{\rm cl}(t)) \, \frac{e^{2\Lambda^2 z_{\rm cl}^2}}{z_{\rm cl}} \, \sqrt{1 + z_{\rm cl}'(t)^2} \, dt \,. \quad (57)$$

The divergence of the path length as the endpoints approach the boundary must be regularized like in Eq. (17), so that

$$G_{\rm reg}^{(FF)}(x, y|\mathcal{C}) = \mathcal{N}' \, \exp[-\Delta \cdot \sigma_{\rm reg}(x, y)/R] \quad (58)$$

Notice that $\Delta \cdot \sigma_{\rm reg}$ is not just a function of the z-coordinate of the classical solution but also the tangent to the classical solution. Δ is the scaling dimension of the gluon field strength and the z-coordinate is dual to the physical extent of field configurations on the boundary. This means the anomalous scaling dimension is not only scale-dependent but is sensitive to rate of change of the physical scale as the particle propagates.

When using a generalized proper time regularization of multidimensional path integrals, care must be taken to handle the multiplicative anomaly, which is the difference between results of the two orders in which one might take the functional and finite determinants in Eq. (43) [34, 35]. Some authors [32, 36] claim either explicitly or implicitly that it is correct to take the functional determinant of $g_{mn}\partial_\lambda^2 + M_{mn}$ before taking the determinant over finite indices, i.e. computing

$$\left(\frac{\det{}' \, {\rm Det}(-g_{mn}\partial_\lambda^2 - M_{mn})}{\det{}' \, {\rm Det}(-g_{mn}\partial_\lambda^2)} \right)^{-1/2} \quad (59)$$

instead of Eq. (43). To evaluate Eq. (59), one can use a multidimensional Gel'fand-Yaglom Theorem [32],

$$\frac{\det{}' \, {\rm Det}(-g_{mn}\partial_\lambda^2 - M_{mn})}{\det{}' \, {\rm Det}(-g_{mn}\partial_\lambda^2)} = \det A(1)^{-1} \quad (60)$$

where $A^m{}_n(\lambda)$ solves

$$(g_{mn}\partial_\lambda^2 + M_{mn}) A^n{}_p(\lambda) = 0 \quad (61)$$

with boundary conditions $A^m{}_n(0) = 0$ and $\dot{A}^m{}_n(0) = \delta^m{}_n$. We will not pursue the details, but using a WKB approximation of this differential equation results in a propagator of the form in Eq. (58) except with

$$\Delta(z_{\rm cl}) = \tfrac{1}{2} R \, {\rm tr} \sqrt{-M_{mn}(z_{\rm cl})} \quad (62)$$

instead of Eq. (54). Ref. [35] shows that the order used in the present paper is the correct order, and gives results in agreement with other non-functional methods. Furthermore, we will see that the use of Eq. (62) does not give a propagator in good agreement with the lattice data.

We can now compare the accuracy of the AdS/QCD calculations with lattice results. We arrive at an explicit expression for $G_{\rm reg}^{(FF)}(x, y|\mathcal{C})$ when we combine Eqs. (52), (54) and (58). For $\mathcal{C}$ we are using the contour shown on the right in Fig. 1, which produces $z_{\rm cl}$ given in Eq. (33). The advantage of this choice is that we can write explicitly

$$\begin{aligned} \cos^2\theta(t) &= \frac{1}{1 + z_{\rm cl}'(t)^2} \\ &= (z_{\rm cl}(t)/z_m)^4 \, \exp[8\Lambda^2 z_m^2 - 8\Lambda^2 z_{\rm cl}(t)^2] \,. \end{aligned} \quad (63)$$

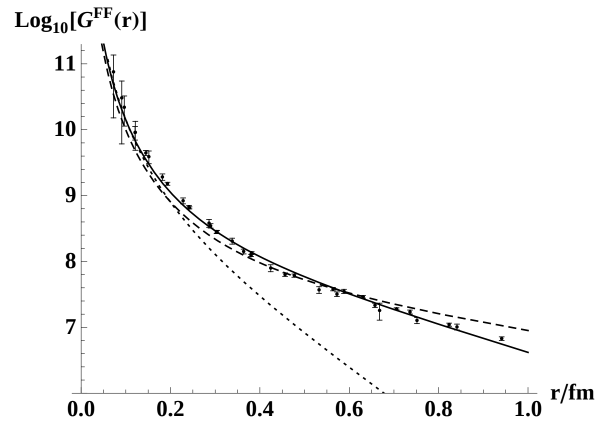

FIG. 3. Boundary-to-boundary field strength correlator with the correct Δ (solid) from Eq. (54) contrasted with the incorrect Δ (dashed) of Eq. (62), and $\Delta = 2$ held constant (dotted). Lattice data was taken from Ref. [31] and error bars from Ref. [37].

Fig. 3 shows the AdS/QCD computation of the propagator with $d = 4$ plotted against the lattice data [31, 37]. The solid curves show the propagator Eq. (58). For contrast, if we use the incorrect ordering of determinants to compute Δ as in Eq. (62), the dashed curve of Fig. 3 results. At small r, the string sits entirely in the near boundary region, as seen in Fig. 2. There, the asymptotically conformal behavior dictates $G(r) \propto 1/r^{2\Delta_{\rm UV}}$, where $\Delta_{\rm UV} = \Delta(z \to 0) = 2$ for the field strength operator. The dotted curve in Fig. 3 shows the propagator computed using Eq. (57) but with the scaling dimension fixed everywhere at the ultraviolet value, $\Delta(z_{\rm cl}(\lambda)) \equiv 2$.

The normalizations of the dashed and dotted curves were set to agree with the lattice data in the small r limit, where the universal $1/r^4$ behavior must hold. For the solid curve, the normalization is determined by minimizing the χ^2 value of the fit. The end result gives $\chi^2/{\rm d.o.f.} \approx 3.07$.

At larger distances, starting around $r = 0.4\,{\rm fm}$, the warp factor has a notable effect on the string's profile. In unwarped AdS and in the small z regime of the metric in Eq. (27), string profiles look simply like rescaled versions of string profiles with the endpoints closer together. Such behavior ceases to hold for large enough r. In fact, soon the string nearly saturates the bound $z_m \leq z_\Lambda = (2\Lambda)^{-1}$. Further increases in r simply extend the portion of the string's profile which sits at the maximum z value. This yields the exponential behavior of the propagator at these

distances,

$$G(x,y) \propto \exp(-|x-y|/\lambda_{\rm gl})\,, \tag{64}$$

where the gluonic correlation length is

$$\lambda_{\rm gl} = R\,(\Delta(z_m)\sqrt{g_{00}(z_m)}\,)^{-1} = (2\sqrt{2}\,\Lambda)^{-1}\,. \tag{65}$$

Substituting $\Lambda \approx 330\,\mathrm{MeV}$, we find that

$$\lambda_{\rm gl} \approx 0.21\,\mathrm{fm}\,. \tag{66}$$

Our result is in good agreement with the lattice results of Ref. [31], which indicate $\lambda_{\rm gl} \approx 0.22\,\mathrm{fm}$. Ref. [38] has $\lambda \approx 0.11 - 0.13\,\mathrm{fm}$, different roughly by a factor of 2.

In this paper, we have addressed a few closely related questions. We set out to compute the two-point function of the gluon field strength operator in the absence of dynamical quarks or other sources of color charge in the fundamental representation. The result is known from quenched lattice QCD computation. To perform this calculation, we first had to gain an understanding of the origin of the scaling dimension formula known from AdS/CFT. From there, we learn how to modify it appropriately for a theory which flows with the physical scale. The results presented here should open up a number of lines of further inquiry, including how to extend to more complicated operators, higher order correlation functions, or making systematic improvements to the approximations contained in this paper.

ACKNOWLEDGMENTS

This work was supported in part by the Director, Office of Science, Office of Nuclear Physics, of the U.S. Department of Energy under grant numbers DE-FG02-05ER41368. The author thanks the Department of Energy's Institute for Nuclear Theory at the University of Washington for its hospitality during the early stages of this work. The author thanks O. Andreev, E. Meggiolaro, C. Morningstar, B. Mueller, R. Plesser and V. Rychkov for fruitful discussion and correspondence, and would like to thank his advisor T. Mehen in particular for asking hard questions.

[1] Juan Martin Maldacena. The Large N limit of superconformal field theories and supergravity. *Adv.Theor.Math.Phys.*, 2:231–252, 1998.
[2] Tadakatsu Sakai and Shigeki Sugimoto. Low energy hadron physics in holographic QCD. *Prog.Theor.Phys.*, 113:843–882, 2005.
[3] Tadakatsu Sakai and Shigeki Sugimoto. More on a holographic dual of QCD. *Prog.Theor.Phys.*, 114:1083–1118, 2005.
[4] Andreas Karch, Emanuel Katz, Dam T. Son, and Mikhail A. Stephanov. Linear confinement and AdS/QCD. *Phys.Rev.*, D74:015005, 2006.
[5] Stanley J. Brodsky and Guy F. de Teramond. Light-Front Dynamics and AdS/QCD Correspondence: The Pion Form Factor in the Space- and Time-Like Regions. *Phys.Rev.*, D77:056007, 2008.
[6] Oleg Andreev and Valentine I. Zakharov. Heavy-quark potentials and AdS/QCD. *Phys.Rev.*, D74:025023, 2006.
[7] Richard C. Brower, Samir D. Mathur, and Chung-I Tan. Glueball spectrum for QCD from AdS supergravity duality. *Nucl.Phys.*, B587:249–276, 2000.
[8] S.S. Gubser, Igor R. Klebanov, and Alexander M. Polyakov. Gauge theory correlators from noncritical string theory. *Phys.Lett.*, B428:105–114, 1998.
[9] Edward Witten. Anti-de Sitter space and holography. *Adv.Theor.Math.Phys.*, 2:253–291, 1998.
[10] Antoine Folacci. Two point functions and stress energy tensors of p forms in De Sitter and anti-De Sitter spaces. *J.Math.Phys.*, 32:2828–2838, 1991.
[11] Iosif Bena. The Propagator for a general form field in AdS(d+1). *Phys.Rev.*, D62:126008, 2000.
[12] Julian S. Schwinger. On gauge invariance and vacuum polarization. *Phys.Rev.*, 82:664–679, 1951.
[13] J.D. Bekenstein and L. Parker. PATH INTEGRAL EVALUATION OF FEYNMAN PROPAGATOR IN CURVED SPACE-TIME. *Phys.Rev.*, D23:2850–2869, 1981.
[14] Christopher R. Stephens. NONPERTURBATIVE BACKGROUND FIELD CALCULATIONS. *Annals Phys.*, 181:120–160, 1988.
[15] Fiorenzo Bastianelli. Path integrals in curved space and the worldline formalism. 2005.
[16] T. Zannias. Path-integral representation of van vleck-dewitt determinant. *Phys. Rev. D*, 28:417–417, Jul 1983.
[17] I.M. Gelfand and A.M. Yaglom. Integration in functional spaces and it applications in quantum physics. *J.Math.Phys.*, 1:48, 1960.
[18] Gerald V. Dunne. Functional determinants in quantum field theory. *J.Phys.A*, A41:304006, 2008.
[19] C. Grosche and F. Steiner. THE PATH INTEGRAL ON THE PSEUDOSPHERE. *Annals Phys.*, 182:120, 1988.
[20] Hiroki Fukutaka and Taro Kashiwa. PATH INTEGRATION ON SPHERES: HAMILTONIAN OPERATORS FROM THE FADDEEV-SENJANOVIC PATH INTEGRAL FORMULA. *Annals Phys.*, 176:301, 1987.
[21] Joseph A. Minahan. Holographic three-point functions for short operators. *JHEP*, 1207:187, 2012.
[22] Boguslaw Broda. NonAbelian Stokes theorem in action. 2000.
[23] J. Smit. Introduction to quantum fields on a lattice: A robust mate. *Cambridge Lect.Notes Phys.*, 15:1–271, 2002.
[24] Juan Martin Maldacena. Wilson loops in large N field theories. *Phys.Rev.Lett.*, 80:4859–4862, 1998.
[25] C D White. The Cornell potential from general geometries in AdS / QCD. *Phys.Lett.*, B652:79–85, 2007.
[26] Stanley Mandelstam. Feynman rules for electromagnetic and Yang-Mills fields from the gauge independent field theoretic formalism. *Phys.Rev.*, 175:1580–1623, 1968.
[27] Yu.M. Makeenko and Alexander A. Migdal. Exact Equation for the Loop Average in Multicolor QCD. *Phys.Lett.*,

B88:135, 1979.

[28] Alexander M. Polyakov and Vyacheslav S. Rychkov. Gauge field strings duality and the loop equation. *Nucl.Phys.*, B581:116–134, 2000.

[29] A. Di Giacomo and H. Panagopoulos. Field strength correlations in the QCD vacuum. *Phys.Lett.*, B285:133–136, 1992.

[30] M. Laine and O. Philipsen. Gauge invariant scalar and field strength correlators in 3-D. *Nucl.Phys.*, B523:267–289, 1998.

[31] A. Di Giacomo, E. Meggiolaro, and H. Panagopoulos. Field strength correlators in QCD at zero and nonzero temperature. *Nucl.Phys.Proc.Suppl.*, 54A:343–347, 1997.

[32] Matt Visser. van Vleck determinants: Geodesic focusing and defocusing in Lorentzian space-times. *Phys.Rev.*, D47:2395–2402, 1993.

[33] Emilio Elizalde, Luciano Vanzo, and Sergio Zerbini. Zeta function regularization, the multiplicative anomaly and the Wodzicki residue. *Commun.Math.Phys.*, 194:613–630, 1998.

[34] T.S. Evans. Regularization schemes and the multiplicative anomaly. *Phys.Lett.*, B457:127–132, 1999.

[35] Antonio Filippi. Chemical potential and multiplicative anomaly. 1998.

[36] J.S. Dowker. On the relevance of the multiplicative anomaly. 1998.

[37] E. Meggiolaro. Private Correspondence.

[38] Gunnar S. Bali, Nora Brambilla, and Antonio Vairo. A Lattice determination of QCD field strength correlators. *Phys.Lett.*, B421:265–272, 1998.